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A method for determining the thermal diffusivity of fluids

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1. INTRODUCTION

ACCURATE measurements of the thermal conductivity of fluids is usually a difficult experimental problem, requiring an accurate measurement of temperature, compensation for heat losses and protection from convection currents caused by buoyancy or surface tension. In this note, we report on a method for directly measuring the thermal diffusivity of fluids which removes some of these problems. In many cases, the diffusivity, rather than the conductivity, is the desired value for computation. In any event, density and heat capacity are relatively easy parameters to determine. The diffusivity is determined by measuring the temperature distribution in a flowing liquid and forcing the measured values to agree with theoretical ones.

The theoretical solution for a liquid with a fully developed velocity profile flowing over an isothermal plate is well known. (The solution for tube flow is given by Bird *et al.* [1].) As far as we know, no experimental data have been published on the temperature profiles in this case. This is primarily because of the experimental difficulties involved if physical probes are used. Optical temperature measurements avoid disturbing the flow field, and also have the benefit of investigating a large portion of the field at once. The method used in this work is holographic interferometry [2]. Only small temperature differences, of the order of 0.5°C, are required. Hence, variations in the physical properties are negligible.

The temperature profile in the system is related to the geometry, fluid velocity and thermal diffusivity, α. If the latter property is unknown, the measured temperature profile may be used to determine the diffusivity. The only requirement of the method is that the fluid be transparent to the light source used, in this case the red light from a helium–neon laser.

2. EXPERIMENTAL SETUP AND PROCEDURES

Water, corn syrup (CS55 ADM, Western Syrup Company) and aqueous solutions of poly(acrylamide) (Dow Separan MG700), from 0.1 to 0.9 wt%, were forced through a long rectangular flow channel. The channel was a plexiglas duct 610 mm long, 200 mm wide and 27 mm high. A flat copper plate was located 300 mm from the inlet to the channel and extended to the exit. The plate could be heated electrically from underneath and was mounted flush with the bottom of the channel. Optical quality glass windows were installed in the sides of the channel near the beginning of the plate.

Viscosities of the corn syrup and the polymer solutions were measured using a rotational viscometer. The corn syrup behaved as a newtonian fluid at all shear rates. The polymer solutions were also newtonian in the shear rate range experienced in the flow channel.

Water flow was by gravity from a feed tank into the channel and then into a receiving tank, where it was pumped back into the feed tank. The corn syrup and polymer solutions were pumped into the channel by a gear pump. Flexible hoses were used to reduce vibration. Mass flow rates were measured by weighing the fluid collected in a given time increment. For the water experiments, the maximum value of the Reynolds number was approximately 10. For the other fluids, this value was always less than 1. The effects of natural convection compared with forced convection may be estimated by the ratio of the Grashof number to the square of the Reynolds number. The maximum value of this parameter in the experiments was approximately 0.25.

A schematic diagram of the optical system is shown in Fig. 1. All the indicated components rested on an optical bench. At the start of the experiment, the fluid was circulated through the system for a time sufficiently long to establish thermal equilibrium (usually of the order of 2 h). A hologram of the flowing fluid in the region of interest, a circle approximately 3 cm in diameter near the beginning of the copper plate, was then made. The light source was a 15 mW He–Ne Spectra Physics laser. The hologram was developed in place. At this point, if the light from the object beam (OB in Fig. 1) was blocked, and the hologram illuminated by the reference beam (RB), an image of the system would appear superimposed on the real channel.

Electrical energy was supplied to the plate. The light from the reconstructed object, at constant temperature, then interfered with the 'real-time' object beam. The result was a set of fringes, indicative of the refractive index difference between the two systems, which could be observed by looking through the holographic plate. When the system came to steady state, as indicated by the constant number and posi-

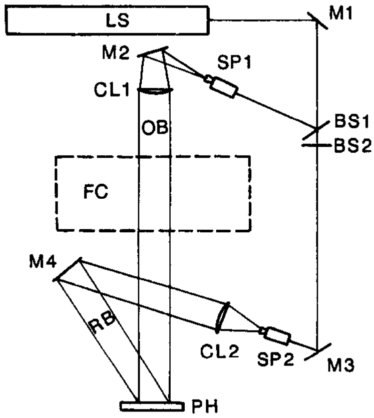

FIG. 1. Schematic diagram of the optical system (BS1, BS2, beam splitters; CL1, CL2, collimating lenses; FC, flow channel; LS, laser light source; M1, M2, M3, M4, plane mirrors; OB, object beam; PH, holographic plate holder; RB, reference beam; SP1, SP2, spatial filters).

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NOMENCLATURE

B	channel height
L	path length
n	refractive index
n'	power-law index
Q	volumetric flow rate
S	fringe shift
T	temperature
W	channel width
x	distance along heated plate
y	distance perpendicular to heated plate.

Greek symbols

α	thermal diffusivity
β	dimensionless parameter
Γ	gamma function
ζ	dummy variable
η	dimensionless position
θ	dimensionless temperature
λ	wavelength of light.

Subscripts

e	value based on assumed diffusivity
ref	reference value
t	value corresponding to measured temperature
W	wall value
∞	far from wall.

tion of the fringes, a photograph of the interference pattern was taken. Further data could be obtained by increasing the flow rate. In general, a new hologram of the system at the higher flow rate was first made before the effects of the heated plate were determined.

The photographic negatives were digitized with an EIKONIXSCAN 78/99 digitizing camera system. The resulting digital images were stored in a DEC VAX 11/780 minicomputer. Subsequently, an image was recalled from memory and displayed and manipulated on a LEXIDATA high resolution monitor to obtain the temperature field.

3. RESULTS AND DISCUSSION

A typical interference pattern is shown in Fig. 2. This is a finite fringe pattern, with the temperature at any point in the flow field directly related to the shift of the fringe from a straight line. In the upper portion of Fig. 2, the fringes are all straight lines, indicating the temperature at these locations is equal to the reference temperature, which is the temperature of the fluid at which the hologram was made. Close to the heated plate in the bottom of the photograph, the temperature of the fluid is greater than that at the reference conditions. The temperature at any point, T, can be calculated from the lateral displacement of the fringe at the point of interest relative to the distance between two unperturbed fringes. This distance, called the fringe shift, S, is related to the temperature by

$$S = (L/\lambda)(\mathrm{d}n/\mathrm{d}T)(T - T_{\mathrm{ref}}) \quad (1)$$

where L is the width of the channel, λ the wavelength of the laser light, n the refractive index, T the temperature and T_{ref}

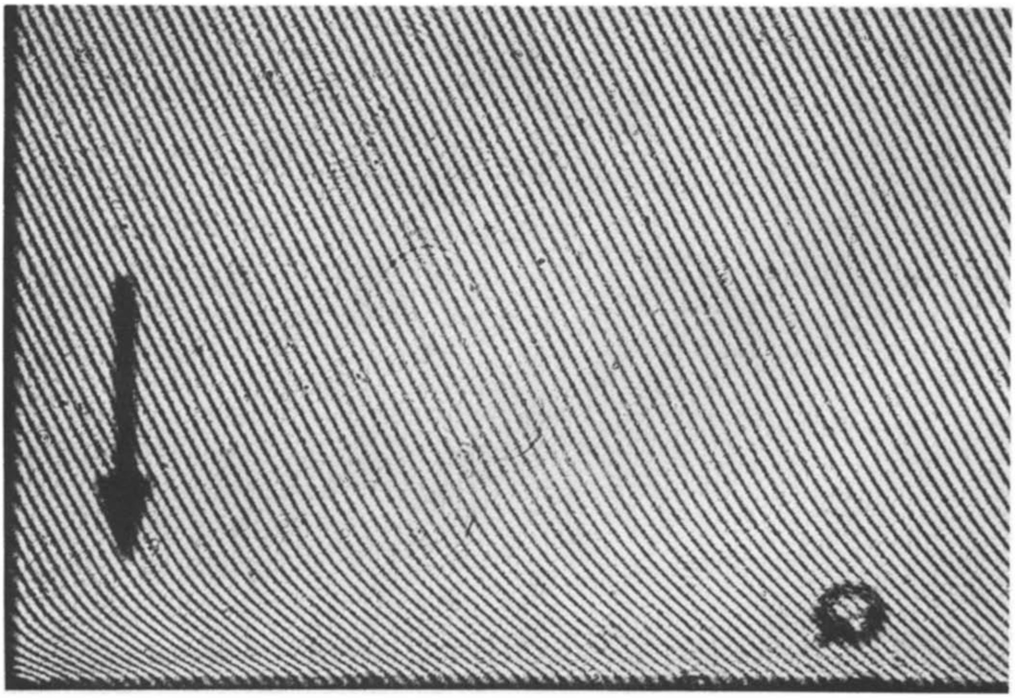

FIG. 2. Steady state finite fringe pattern in water above an isothermal plate near the leading edge. The flow is left to right.

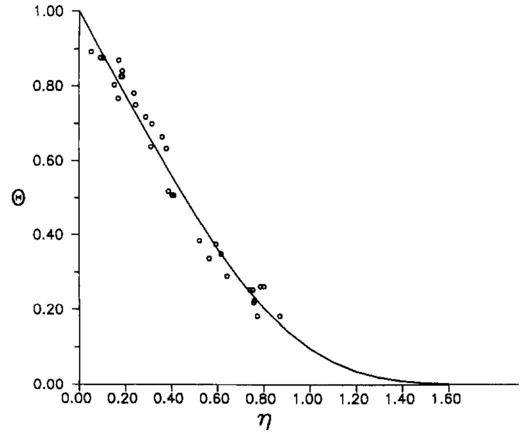

FIG. 3. Comparison of theoretical and experimental dimensionless temperature profiles for water above a flat plate. The experimental points are based on an average measured value for $\alpha = 1.46 \times 10^{-7}$ m^2 s^{-1}.

the temperature of the system at the reference conditions. Close inspection of the fringe shift at the surface of the plate indicates that there is a 3–5% variation in the shift over the surface. The assumption of an isothermal plate is a good one.

The theoretical temperature distribution may be found using the method of Bird *et al.* [1]. We assume the fluid may be modeled by the power law. With the assumption of negligible heat penetration into the bulk of the fluid, the dimensionless temperature rise at any point is

$$\theta = [1/\Gamma(4/3)] \int_{\eta}^{\infty} \exp(-\zeta^3)\, \mathrm{d}\zeta \qquad (2)$$

where θ is the dimensionless temperature rise

$$\theta = (T - T_\infty)/(T_\mathrm{W} - T_\infty) \qquad (3)$$

η the dimensionless position relative to the heated plate

$$\eta = y/(9\beta x)^{1/3} \qquad (4)$$

and ζ a dummy variable of integration.

The parameter β is given by

$$\beta = \alpha W B^2 n'/[2Q(1+2n')]. \qquad (5)$$

In these equations, x and y are the coordinates of the point of interest relative to the surface of the leading edge of the heated plate, B the channel height, Q the volumetric flow rate, and n' the power law index. The subscripts W and ∞ refer to the plate and to the fluid far away. As is apparent from equations (2) and (3), it is not necessary to compute the actual temperature at any location. Rather, the quantity of interest is the local temperature rise relative to that at the surface of the plate. In terms of the fringe shift

$$\theta = S/S_\mathrm{W}. \qquad (6)$$

Table 1. Thermal diffusivities

Material	Thermal diffusivity (m^2 s^{-1})
Water	1.46×10^{-7}
Corn syrup	1.72×10^{-7}
Polymer solutions (wt%):	
0.11%	1.35×10^{-7}
0.28%	1.23×10^{-7}
0.43%	1.06×10^{-7}
0.60%	1.00×10^{-7}
0.90%	0.96×10^{-7}

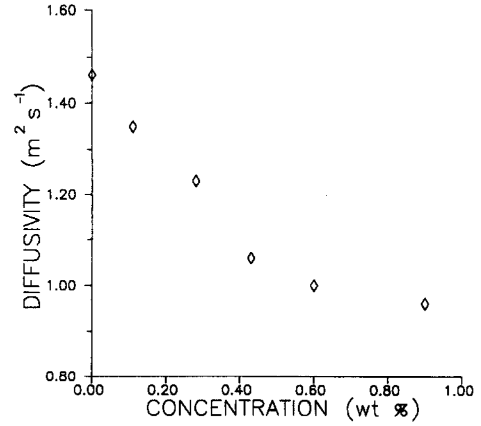

FIG. 4. Measured variation of the thermal diffusivity of poly(acrylamide) solutions with concentration.

The determination of the thermal diffusivity proceeds in the following manner. The value of θ is measured at a certain location (x, y). From the theoretical temperature distribution, equation (2), this corresponds to a certain value of the parameter η, η_t. At this same location, using an assumed value of the thermal diffusivity and the known values of Q and n, an 'experimental' value of η, η_e, may be calculated. The 'best' value of α is the one which minimizes the sum of the squared errors, $\Sigma_i(\eta_t - \eta_e)_i^2$, for a number of observations. In practice, we use 50 or so different locations for a particular fluid.

Figure 3 compares the theoretical temperature profile with representative experimental data for water with α computed in this manner. The experimental value of α was found to be 1.46×10^{-7} m^2 s^{-1}. The literature value at 20°C is 1.42×10^{-7} m^2 s^{-1} [3]. The difference between the two values is less than 3%. The repeatability of the method, as determined by starting with a completely different hologram, is better than 2%.

For the corn syrup, a value of $\alpha = 1.72 \times 10^{-7}$ was computed. The results for the polymer solutions are illustrated in Fig. 4. It is interesting to note that the diffusivity drops off fairly quickly with increasing polymer concentration, and above about 1% polymer, becomes independent of the concentration. The data are summarized in Table 1.

4. CONCLUSIONS

Holographic interferometry has been used to measure the temperature distributions above an isothermal plate. The temperatures are in good agreement with theoretical predictions. The measured data may be used to obtain the thermal diffusivity of the fluid by forcing the measurements to agree with the theory. The thermal diffusivity of water found in this way was about 3% higher than literature values.

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